Subject Code : | 1 | 6 | 6 |

(English Version)

Subject : Higher Mathematics 2nd Paper

Time : 3 Hours **Full Marks — 75**

[The figure in the right margin indicate full marks]

Group-A: Algebra

1. Answer any **two** of the followings questions: $5 \times 2 = 10$
   a. For all a, b $\in \mathbb{R}$, show that $|a + b| \le |a| + |b|$
   b. Solve and show the solution set on real line of $\frac{1}{|3x - 5|} > 2$.
   c. If $\sqrt[3]{a + ib} = x + iy$, then show that $4(x^2 - y^2) = \frac{a}{x} + \frac{b}{y}$.

2. Answer any **two** of the following questions: $5 \times 2 = 10$
   a. If one of the roots of $ax^2 + bx + c = 0$ is the square of the other, then prove that $c(a - b)^3 = a(c - b)^3$
   b. Show that, in the expression of $(1 - 2x)^{-\frac{1}{2}}$ the coefficient of $(r + 1)$th term is $\frac{(2r)!}{(r!)^2 . 2^r}$
   c. Show that, the middle term in the expansion of $\left(x - \frac{1}{x}\right)^{2n}$ is $\frac{1.3.5 \ldots (2n - 1)}{n!}(-2)^n$

Group-B: Geometry

3. Answer any **two** of the following questions: $5 \times 2 = 10$
   a. Find the equation of the parabola whose vertex is the point (4, −3), directrix parallel to the x-axis and which passes through the point (−4, −7)
   b. Find the length of the latus rectum, eccentricity and the co-ordinates of the two foci of the ellipse $2x^2 + 3y^2 - 1 = 0$
   c. Find the equation of the hyperbola whose directrix is $2x + y = 1$, coordinates of the focus (1, 1) and the eccentricity is $\sqrt{3}$.

Group-C: Trigonometry

4. Answer any **two** of the following questions: $5 \times 2 = 10$
   a. Prove that $\sin^{-1}(\sqrt{2}\sin\theta) + \sin^{-1}(\sqrt{\cos 2\theta}) = \frac{\pi}{2}$
   b. Find the value of $\sin\theta - 2 = \cos 2\theta$, when, $-2\pi < \theta < 2\pi$.
   c. Solve the equation $\cos\theta - \cos 7\theta = \sin 4\theta$

Gropu-D: Mechanics

5. Answer any **one** of the following questions: $5 \times 2 = 10$
   a. (i) Find the resultant and the point of action of the two unequal unlike parallel forces acting on a rigid body.
   (ii) A uniform plank of length 2a and weight W is suspended horizontally on two vertical props at a distance b apart. The greatest weights that can be placed at the two ends in succession without upsetting the plank are W_1 and W_2 respectively. Show that $\frac{W_1}{W + W_1} + \frac{W_2}{W + W_2} = \frac{b}{a}$
   b. (i) State the varignon's theorem and prove it when the forces are intersects at a point on the rigid body.
   (ii) The resultant of two like parallel forces P and Q passes through a point O, when P is increased by R and Q by S, the resultant still passes through O and also when Q, R replace by R and Q respectively. Prove that $S = R - \frac{(Q - R)^2}{P - Q}$.

6. Answer any **one** of the following questions: $5 \times 2 = 10$
   a. (i) Under usual notation, prove that $v^2 = u^2 + 2fs$.
   (ii) A particle is projected at an angle 45° from a point at a distance x from the foot of the vertical wall. It just clears the wall and fall on the ground at a distance y on the other side of the wall. Show that the height of the wall is $\frac{xy}{x + y}$.
   b. (i) Prove that the equation of the path of a projectile in vacuo is $y = x\tan\alpha\left(1 - \frac{x}{R}\right)$, where α is the angle of projection and R is the horizontal range.
   (ii) A stone falling from the top of a vertical tower has descended x meters when another is let fall from a point y meters below the top. If they fall from rest and reach the ground together, show that the height of the tower is $\frac{(x + y)^2}{4x}$.

Group-E: Linear Programming

7. Solve the linear Programming with help of graph and maximize $Z = 3x + 4y$.
   Under the conditions : $x + y \le 450$, $2x + y \le 600$, $x \ge 0$, $y \ge 0$. 5
   Or, One business centre produces two goods A and B and profits Tk. 3 and Tk. 4 per unit of goods respectively. Each of the goods is prepared by two machines M_1 and M_2. Machine M_1 and M_2 take time to prepare goods A in 1 and 2 minutes respectively. Goods B is prepared by machine M_1 and M_2 in 1 and 1 minute respectively. M_1 and M_2 machines are to be used $7\frac{1}{2}$ hours and 10 hours in every working day respectively. In order to get highest profit how much of the goods A and B should be produced? Prepare the linear programming.

Group-F: Statistics

Answer any two of the following questions: $5 \times 2 = 10$

8. a. Determine the Mean deviation and Standard deviation from the following frequency distribution table below : 5

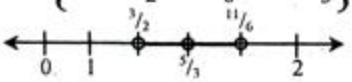
60-62	63-65	66-68	69-71	72-74
5	18	42	27	8

b. State and prove that the Addition rule of probability for the event mutually not exclusive.
c. Out of 200 candidates in an examination in Mathematics and Statistics, 20 fail in Statistics, 40 fail in Mathematics and 10 fail in both the subjects. If a candidate is selected at random then what will be the probability of his passing in Statistics and failed in Mathematics.

Answer

1. (b) Solve : $\frac{3}{2} < x < \frac{11}{6}$ But $x \ne \frac{5}{3}$

   Solution Set : $S = \left\{x \in \mathbb{R} : \frac{3}{2} < x < \frac{11}{6} \text{ and } x \ne \frac{5}{3}\right\}$

   Real line : 

3. (a) $x^2 - 8x + 16y + 64 = 0$
   (b) $\frac{2\sqrt{2}}{3}$, $e = \frac{1}{\sqrt{3}}$, $\left(\pm\frac{1}{\sqrt{6}}, 0\right)$
   (c) $7x^2 - 2y^2 + 12xy - 2x + 4y - 7 = 0$

4. (b) $\theta = -\frac{3\pi}{2}, \frac{\pi}{2}$
   (c) $\theta = \frac{n\pi}{4}, \frac{n\pi}{3} + (-1)^n\frac{\pi}{18}$, when $n \in \mathbb{Z}$.

7. (a) $Z_{max} = 1800$
   Or, A = 150 and B = 300

8. (i) 2.265 inchi and 2.92 inchi
   Or, (iii) $\frac{3}{20}$