

## (English Version)

## Subject : Higher Mathematics 2nd Paper

Time : 3 Hours

Full Marks — 75

[ The figure in the right margin indicate full marks]

## Group - A (Algebra)

1. Answer any two of the following questions:  $5 \times 2 = 10$
- For all  $a, b \in \mathbb{R}$ . Prove that  $|a + b| \leq |a| + |b|$
  - If  $|y - 1| < \frac{1}{10}$  show that,  $|y^2 - 1| < \frac{21}{100}$
  - If  $\omega$  is a cube root of unity, show that  
 $(-1 + \sqrt{-3})^4 + (-1 - \sqrt{-3})^4 = -16$
2. Answer any two of the following questions:  $5 \times 2 = 10$
- If the roots of the equation :  $x^2 - bx + c = 0$  and  $x^2 - cx + b = 0$  differ only by a constant then prove that  $b + c + 4 = 0$
  - Find the coefficient of  $x^{10}$  in the expansion  $(2x^2 - \frac{3}{x})^{11}$
  - If  $y = 2x + 3x^2 + 4x^3 + \dots \dots \dots \alpha$ , show that  $x = \frac{1}{2}y - \frac{3}{8}y^2 + \frac{5}{16}y^3 \dots \dots \dots \alpha$

## Group - B (Geometry)

3. Answer any two of the following questions:  $5 \times 2 = 10$
- The parabola  $y^2 = 4px$  passes through the point  $(3, -2)$ . Find its latus rectum and the coordinates of the focus.
  - Find the equation of the ellipse whose focus  $(3, 4)$  directrix  $x + y - 2 = 0$  and eccentricity is  $\frac{1}{3}$ .
  - Find the coordinates of the foci and equations of the directrices of the hyperbola  $\frac{x^2}{9} - \frac{y^2}{16} = 1$

## Group-C (Trigonometry)

4. Answer any two of the following questions:  $5 \times 2 = 10$
- Show that,  $\sec^2(\tan^{-1}2) + \operatorname{cosec}^2(\cot^{-1}3) = 15$
  - Solve :  $\sin\theta + \cos\theta = \sqrt{2}$ ,  $-\pi < \theta < \pi$
  - $\tan^{-1}\frac{2}{3} = \frac{\pi}{2} - \sec^{-1}\frac{\sqrt{13}}{2}$

## Group - D (Statics and Dynamics)

5. Answer any two of the following questions:  $5 \times 2 = 10$
- State and prove the law of triangle of forces.
  - P, Q are two parallel forces. If two equal and opposite forces along any two parallel lines at a distance b apart in

the plane of P, Q are combined with them. Show that the resultant displaced by a distance  $\frac{bS}{P+Q}$ .

6. Answer any one of the following questions:  $5 \times 1 = 5$
- Find the magnitude, direction and the line of action of the resultant of two unequal and like parallel forces.
  - Three equal forces P act at a point parallel to the sides BC, CA and AB of a triangle ABC. Prove that their resultant is given by  $P\sqrt{3 - 2\cos A - 2\cos B - 2\cos C}$ .
7. Answer any one of the following questions:  $5 \times 1 = 5$
- Determine the greatest height and the time to reach the highest point of a particle projected with the velocity u at an angle  $\alpha$  to the horizon.
  - A particle is projected vertically upwards and it takes t second to rise to a height h. If it takes t' second to rise to a height h. Prove that  $h = \frac{1}{2}gtt'$ .

8. Answer any one of the following questions:  $5 \times 1 = 5$
- Under usual notations, deduce  $v^2 = u^2 + 2fs$ .
  - the resultant of two velocities u and v acting at a point is w. The resolved part of w along the direction of u is v. Show that the angle between the two velocities is  $\cos^{-1}\frac{v-u}{v}$  and  $w = \sqrt{v^2 - u^2 + 2uv}$ .

## Group - E (Linear programming)

9. (a) What is the linear programming Write down the conditions of linear programming.  
 (b) Solve the following programming with the help of graph and minimize of  $z = 2y - x$ .  
 Conditions :  $3y - x \leq 10$ ,  $x + y \leq 6$ ,  $x - y \leq 2$ ,  $x, y \geq 0$

## Group - F (Statistics)

10. Answer any two of the following questions:  $5 \times 2 = 10$
- State and prove the multiplication law of probability for two independent events.
  - Find the variance for the first n natural numbers.
  - If  $P(A) = \frac{1}{3}$ ,  $P(B) = \frac{3}{4}$ , if A and B are independent events then find  $P(A \cap B)$  and  $P(A \cup B)$

2. (b)  ${}^{11}C_4 \cdot 2^7 \cdot 3^4$
3. (a)  $\frac{4}{3}, (\frac{1}{3}, 0)$   
 (b)  $17x^2 - 2xy + 17y^2 - 104x - 140y + 446 = 0$   
 (c)  $(\pm 5, 0); 5x + 9 = 0, 5x - 9 = 0$
4. (b)  $\theta = \frac{\pi}{4}$

7. (a)  $H = \frac{u^2}{2g} \sin^2\alpha, T_1 = \frac{u}{g} \sin\alpha$
10.  $Z_{\min} = -2$
12.  $\frac{n^2 - 1}{12}$
13.  $\frac{1}{4}$  and  $\frac{5}{6}$