Subject Code : | 1 | 6 | 6 |

(English Version)

Subject : Higher Mathematics 2nd Paper

Time : 3 Hours **Full Marks — 75**

[The figure in the right margin indicate full marks]

1. Answer any **two** of that following questions : $5 \times 2 = 10$

a. If $a, b \in \mathbb{R}$, Prove that $|a + b| \leq |a| + |b|$

b. Find the value of $\sqrt[3]{1}$ prove that the sum of the three cube roots of units is zero.

c. If $\sqrt[3]{a + ib} = x + iy$ show that $\sqrt[3]{a - ib} = x - iy$

2. Answer any **one** of the following questions : $5 \times 1 = 5$

a. Define linear programming. what are condition and advantages of the linear programming.

b. Solve the following linear programming with the help of graph and find the maximum value,
$Z_{max} = 3x + 4y$, where $x + y \leq 7, 2x + 5y \leq 20, x > 0, y > 0$.

3. Answer any **two** of the following questions : $5 \times 2 = 10$

a. If one of the root of the equation $27x^2 + 6x - (p + 2) = 0$ is the square of the other, Find the value of P.

b. Find the term independent of x in the expression of $\left(2x - \frac{1}{4x^2}\right)^{12}$.

c. If $y = x + x^2 + x^3 + x^4 + \ldots\ldots$, show that $x = y - y^2 + y^3 - y^4 + \ldots\ldots$

4. Answer any **two** of the following questions : $5 \times 2 = 10$

a. Find the equation of the parabola whose focus is the point $(-1, 1)$ and the equation of whose directrix is $x + y + 1 = 0$.

b. For what value of P does the ellipse $\frac{x^2}{p} + \frac{y^2}{5^2} = 1$ passes through the point $(6, 4)$? Find the eccentricity and the coordinates of the foci of the ellipse.

c. Find the length of the axes, eccentricity and coordinates of the centre of the hyperbola $x^2 - 3y^2 - 2x = 8$

5. Answer any **two** of the following questions : $5 \times 2 = 10$

a. Prove that, $2\tan^{-1} x = \tan^{-1} \frac{2x}{1 - x^2} = \sin^{-1} \frac{2x}{1 + x^2} = \cos^{-1} \frac{1 - x^2}{1 + x^2}$

b. If $\sin^{-1} x + \sin^{-1} y = \frac{\pi}{2}$, show that $x^2 + y^2 = 1$

c. Solve: $\sin x + \cos x = 1$

6. a. State and prove Lamis theorem. 5+5=10

b. Forces P and Q acting at a point O have the resultant R. If a straight line meet their lines of action at L. M and N respectively, Prove that, $\frac{P}{OL} + \frac{Q}{OM} = \frac{R}{ON}$

OR, a. Find the magnitude direction and point of application of the resultant of two unequal like parallel.

b. P and Q are two like parallel forces. If p is moved parallel to itself through a distance x, show that the resultant of P and Q will move through distance $\frac{Px}{P + Q}$

7. a. With usual notations prove that, $s = ut + \frac{1}{2}ft^2$. 5+5=10

b. The greatest resultant velocity is n times of the least resultant velocity of two velocities. If the resultant velocity be the half of the algebraic sum of these two velosities and the angle between them be α, then show that, $\cos\alpha = \frac{n^2 + 2}{2(1 - n^2)}$

OR, a. Prove that, the path of a body projected in vacuo is a parabola.

b. A cricket ball is thrown by Masrafi from a height 2 metres above the ground with a velocity of 20 m/sec. at an angle 30° to the horizon. Another player Musfique caught it at a height of 1 metre above the ground. How far apart were the two players?

8. Answer any **two** of the following questions : $5 \times 2 = 10$

a. Find the variance for the n natural numbers.

b. State and prove the law of additions of probabilities for exclusive events.

c. The probabilities of A and B for solving a problem are $\frac{1}{3}$ and $\frac{1}{4}$ respectively. Find the probability of their solving it together.

Answer

1. (b) $1, \frac{-1 + \sqrt{-3}}{2}, \frac{-1 - \sqrt{-3}}{2}$
2. (b) 23
3. (a) $6, -1$
   (b) 495
4. (a) $(x - y)^2 + 2x - 6y + 3 = 0$
   (b) $100; \frac{\sqrt{3}}{2}; (\pm 5\sqrt{3}, 0)$
   (c) $6, 2\sqrt{3}; \frac{2}{\sqrt{3}}; (1, 0)$
5. (c) $2n\pi, (4n + 1)\frac{\pi}{2}$, where n is zero or any integer.
7. Or, (b) 37.07 m (approx)
8. (a) $\frac{n^2 - 1}{12}$
   (c) $\frac{1}{2}$