

## (English Version)

## Subject : Higher Mathematics 2nd Paper

Time : 3 Hours

Full Marks — 75

[ The figure in the right margin indicate full marks]

1. Answer any two of the following questions :  $5 \times 2 = 10$
- Prove that,  $\sqrt{3}$  is an irrational number.
  - If  $a^2 + b^2 = 1$ , show that a real value of  $x$  will satisfy the equation  $\frac{1-ix}{1+ix} = a-ib$ , when  $a$  and  $b$  are real numbers.
  - If  $a$  and  $b$  are real numbers, prove that  $|a-b| \geq |a|-|b|$
2. Define linear programming. Write down the steps for the formation of the linear problems. 5
- Or, Solve the following linear programming with the help of graph and maximize  $z = 3x + 4y$ , constrains  $x + y \leq 7$ ,  $2x + 5y \leq 20$ ,  $x, y \geq 0$ .
3. Answer any two of the following questions:  $5 \times 2 = 10$
- If one of the roots of the equation  $27x^2 + 6x - (p+2) = 0$  is the square of the other, find the value of  $p$ .
  - If the roots of the equation  $x^2 - bx + c = 0$  and  $x^2 - cx + b = 0$  differ only by a constant then prove that  $b + c + 4 = 0$ .
  - If the coefficients of  $x^7$  and  $x^8$  are equal in the expansion of  $\left(3 + \frac{x}{2}\right)^n$ , where  $n \in \mathbb{N}$ , find the value of  $n$ .
4. Answer any two of the following questions:  $5 \times 2 = 10$
- The focal distance of any point on the parabola is  $y^2 = 8x$ ; find the coordinates of the point.
  - Find the equation of the directrix  $x$  of the parabola whose focus is  $(3, 4)$  and whose vertex is  $(0, 0)$
  - For what value of  $p$  does the ellipse  $\frac{x^2}{p} + \frac{y^2}{5} = 1$  pass through the point  $(6, 4)$ ? Find the eccentricity and the coordinates of the foci of the ellipse.
5. Answer any two of the following questions:  $5 \times 2 = 10$
- Prove that,  $\sin^{-1}(\sqrt{2} \sin \theta) + \sin^{-1}(\sqrt{\cos 2\theta}) = \frac{\pi}{2}$
  - Prove that,  $\operatorname{cosec}^{-1} \tan \cos^{-1} x = x$
  - Solve the equation,  $\cot \theta + \tan \theta = 2 \sec \theta$ ,  $-\pi < \theta < 2\pi$
6. Answer any two of the following questions:  $5 \times 2 = 10$
- Under usual notations deduce  $v^2 = u^2 + 2fs$ .
  - If  $v_1, v_2, v_3$  be the average velocities in three successive intervals of times  $t, t_2, t_3$  respectively of the point moving on the straight line with uniform acceleration. Show that  $\frac{v_1 - v_2}{v_2 - v_3} = \frac{t_1 + t_2}{t_2 + t_3}$
- Or, a. Show that the path of a projectile in vacuou in a parabola.
- b. If  $t$  be the time taken by a projectile to reach at the point  $P$ , it also taken more  $t'$  seconds to reach the plane projection. Show that the height of the point  $P$  is  $\frac{1}{2} g t t'$ .
7. Answer any two of the following questions:  $5 \times 2 = 10$
- The algebraic sum of the resolved parts of two forces acting at a point in any direction is equal to the resolved parts of their resultant in the same direction.
  - The resultant of two concurrent forces  $p$  and  $Q$  ( $P > Q$ ) trisects the angle between them. Show that, the angle between them is  $3 \cos^{-1} \left( \frac{P}{2Q} \right)$  and the resultant equal to  $\frac{P^2 - Q^2}{Q}$ .
- Or, a. Find magnitude and point of action of the resultant of two like parallel forces acting on a rigid body.
- b.  $P$  and  $Q$  are two like parallel forces. If  $p$  is moved parallel to itself through a distance. Show that the resultant of  $p$  and  $Q$  will move through distance  $\frac{P x}{P + Q}$ .
8. Answer any two of the following questions :  $5 \times 2 = 10$
- State and prove the law of addition rule for two not mutually exclusive events.
  - A box contains 6 white, 7 red and 9 black balls of different shape. A ball is drawn at random. What is the probability of the ball to be red or white?
  - Find the standard deviation from the following frequency distribution table:

values $x_i$	10	13	25	30	37	42	45
Frequencies $f_i$	3	7	8	15	10	5	2

2. OR, 23

3. (a) 6, -1

(c) 55

4. (a)  $(6, \pm 4\sqrt{3})$ (b)  $3x + 4y + 25 = 0$ (c)  $100, \frac{\sqrt{3}}{2}, (\pm 5\sqrt{3}, 0)$ 5. (c)  $-\frac{11\pi}{6}, -\frac{7\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6}$ 8. (b)  $\frac{13}{22}$ 

(c) 10 (approx)