

(English Version)

Subject : Higher Mathematics 2nd Paper

Time : 3 Hours

Full Marks — 75

[The figure in the right margin indicate full marks]

1. Answer any **two** of the following questions: $5 \times 2 = 10$
- For all $a, b \in \mathbb{R}$, Prove that $|a + b| \leq |a| + |b|$
 - Find the square root of $2i$
 - If $(a\omega^2 + b + c\omega)^3 + (a\omega + b + c\omega^2)^3 = 0$, show that, either $a = \frac{1}{2}(b + c)$ or $b = \frac{1}{2}(c + a)$ or $c = \frac{1}{2}(a + b)$.
2. Answer any **one** of the following questions: $5 \times 1 = 5$
- Find graphically the minimum value of $z = 2x - y$. Constraints: $x + y \leq 5$, $x + 2y \leq 8$, $4x + 3y \geq 12$, $x, y \geq 0$.
 - A man wants to purchase pens and pencils with an amount not exceeding Tk. 100. Each pen costs Tk. 12 and each pencil costs Tk. 8. Find how many of each kind he can purchase so that under the condition that he will purchase at least one pen and at most 8 pencils so that the total number may be maximum.
3. Answer any **two** of the following questions: $5 \times 2 = 10$
- If one of the roots of $ax^2 + bx + c = 0$ is double of one of the roots of $cx^2 + bx + a = 0$ then show that either $2a = c$ or, $(2a + c)^2 = 2b^2$
 - If α, β, γ are the roots of the equation $x^3 - px^2 + qx - r = 0$, find the value of $\sum \frac{1}{\alpha^2}$.
 - If in the expansion of $(a + 3x)^n$ the first three terms are respectively $b, \frac{21}{2}bx$ and $\frac{189}{4}bx^2$, find the value of a, b and n .
4. Answer any **two** of the following questions: $5 \times 2 = 10$
- Find the vertex, focus, latus rectum and equation of the axis and directrix of the parabola $5x^2 + 30x + 2y + 59 = 0$
 - Taking the major and minor axes as the x and y -axes find the equation of the ellipse whose eccentricity = $1/3$ and length of latus rectum = 8.
 - Find the equation of the hyperbola, equation of whose directrix is $2x + y = 1$, coordinates of the focus $(1, 1)$ and eccentricity is 3.
5. Answer any **two** of the following questions: $5 \times 2 = 10$
- Prove that, $\cos^{-1} \frac{1}{\sqrt{5}} - \frac{1}{2} \sin^{-1} \frac{3}{5} + \tan^{-1} \frac{1}{3} = \tan^{-1} 2$.
 - Solve : $\cos \theta - \cos 7\theta = \sin 4\theta$
 - Solve : $\cos x + \sqrt{3} \sin x = \sqrt{2}$
6. Answer any **one** of the following questions: $5 \times 1 = 5$
- The resultant of two concurrent forces P and Q ($P > Q$) trisects the angle between them. Show that, the angle between them is $3\cos^{-1} \left(\frac{P}{2Q} \right)$ and the resultant equal to $\frac{P^2 - Q^2}{Q}$.
7. Answer any **one** of the following questions: $5 \times 1 = 5$
- State and prove Varignon's Theorem (When the two forces are concurrent)
 - P and Q are like parallel forces. If a couple (S, b) in the plane of P, Q is combined with them, show that the resultant is displaced through a distance $\frac{Sb}{P+Q}$.
8. Answer any **one** of the following : $5 \times 1 = 5$
- Find the magnitude and direction of the resultant of two given velocities inclined at an angle α to each other.
 - If a point moving under acceleration describes successive equal distances in time t_1, t_2, t_3 , then prove that, $\frac{1}{t_1} + \frac{1}{t_2} + \frac{1}{t_3} = \frac{3}{t_1 + t_2 + t_3}$.
9. Answer any **one** of the following questions: $5 \times 1 = 5$
- A stone is dropped into a well and the sound of the splash is heard in t sec. If v be the velocity of sound and h be the depth of the well, prove that,
 - $t = \sqrt{\frac{2h}{g}} + \frac{h}{v}$
 - $gv^2t^2 - 2ghvt + h(gh - 2v^2) = 0$
 - A particle is projected with an initial velocity u . If the greatest height attained by the particle, H . Prove that the range R on the horizontal plane through the point of projection is $R = 4\sqrt{H \left(\frac{u^2}{2g} - H \right)}$.
10. Answer any **two** of the following questions: $5 \times 2 = 10$
- State and prove the law of addition of probabilities for exclusive events.
 - A box contains 5 red and 4 white balls and another box contains 3 red and 6 white balls. One ball is drawn at random from each box. Find the probability that at least one of the two balls to be red.
 - Find the Variance from the following frequency distribution table:

values x_i	10	13	25	30	37	42	45
frequencies f_i	3	7	8	15	10	5	2

1. (b) $\pm(1 + i)$
2. (a) $Z_{\min} = -4$
(b) 3 pens, 8 pencils.
3. (b) $\frac{1}{r^2}(q^2 - 2pr)$
(c) 2, 128, 7
4. (a) $(-3, -7), \left(-3, -\frac{71}{10}\right); \frac{2}{5}, x + 3 = 0; 10y + 69 = 0$
(b) $\frac{4x^2}{81} + \frac{y^2}{18} = 1$
(c) $31x^2 + 4y^2 - 26x + 8y + 36xy - 1 = 0$
5. (b) $\theta = \frac{n\pi}{4}, \frac{n\pi}{3} + (-1)^n \frac{\pi}{18}$, where n is 0 or any integer.
(c) $x = 2n\pi + \frac{7\pi}{12}, 2n\pi + \frac{\pi}{12}$, where n is 0 or any integer.
8. (a) $w = \sqrt{u^2 + v^2 + 2uv \cos \alpha}$, $\theta = \tan^{-1} \left(\frac{v \sin \alpha}{u + v \cos \alpha} \right)$
10. (b) $\frac{19}{27}$
(c) 100