

[ The figure in the right margin indicate the full works. ]

## Group-A (Algebra)

1. Answer any two of the following questions :
- $5 \times 2 = 10$

a. Define identity matrix. If  $A = \begin{bmatrix} 3 & -4 & 2 \\ -2 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix}$  and

$$B = \begin{bmatrix} 1 & 2 & -2 \\ 2 & 5 & -4 \\ 3 & 7 & -5 \end{bmatrix}$$
 show that,  $AB = BA = I_3$ .

b. If  $A = \begin{bmatrix} 2 & -1 & -1 \\ 1 & -2 & 1 \\ 1 & -1 & 2 \end{bmatrix}$ , determine  $\text{Adj}(A)$  and then  $A^{-1}$  if it is exist.c. Prove that,  $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 - 1 & b^3 - 1 & c^3 - 1 \end{vmatrix} = (abc - 1)(a - b)$ 

(b - c)(c - a), also write down a condition to find the value of the determinant without expansion.

2. Answer any one of the following questions:
- $5 \times 1 = 5$

a. How many triangles can be formed by joining the angular points of a plane figure of 20 sides? How many diagonals the figure has?

b. To find the number of permutation of n different things taking r at a time [n and r are positive integers and  $n \geq r$ ]

## Group-B (Trigonometry)

3. Answer any two of the following questions :
- $5 \times 2 = 10$

a. If  $(a^2 - b^2)\sin x + 2ab\cos x = a^2 + b^2$  and x is an acute positive angle, then find the value of  $\tan x$  and  $\text{cosec} x$ .b. Draw the graph of  $y = \cos 2x$ , when  $0 \leq x \leq 2\pi$ .c. If  $\tan \theta = \frac{5}{12}$  and  $\sin \theta$  is negative, find the value of

$$\frac{\sin \theta + \cos(-\theta)}{\sec(-\theta) + \tan \theta}$$

4. Answer any two of the following questions :
- $5 \times 2 = 10$

a. Prove that,  $\sin^3 x + \sin^3(120^\circ + x) + \sin^3(240^\circ + x) = -\frac{3}{4} \sin 3x$ .b. If  $A + B + C = \frac{\pi}{2}$ , prove that,  $\sin^2 A + \sin^2 B + \sin^2 C + 2\sin A \sin B \sin C = 1$ c. Prove that, in any triangle,  $a(\cos C - \cos B) = 2(b - c) \cos^2 \frac{A}{2}$ .

## Group-C (Geometry)

5. Answer any three of the following questions :
- $5 \times 3 = 15$

a. Show that, origin is a point of trisection of the straight line joining the points (-3, -2) and (6, 4). Find the coordinates of the other point of trisection.

b. Find the equation of the straight line parallel to the y-axis and passing through the point of intersection of the straight lines  $2x - 7y + 11 = 0$  and  $x + 3y - 8 = 0$ .c. Find the coordinates of the foot of the perpendicular drawn from the point (2, -1) to the straight line  $3x - 4y + 5 = 0$ d. Show that the straight line  $x - 3y = 5$  is a tangent of the circle  $x^2 + y^2 - 6x + 8y + 15 = 0$ . Find the equation of the diameter which passes through the point of contact.

6. Answer any one of the following questions :
- $5 \times 1 = 5$

a. Show that, the vectors  $\underline{a} = 3\hat{i} - 2\hat{j} + \hat{k}$ ,  $\underline{b} = \hat{i} - 3\hat{j} + 5\hat{k}$ ,  $\underline{c} = 2\hat{i} + \hat{j} - 4\hat{k}$  form a right angled triangle.

b. Prove by the vector method that the medians of a triangle are concurrent.

## Group-D (Calculus)

7. If
- $f(x) = \ln(\sin x)$
- and
- $\phi(x) = \ln(\cos x)$
- , show that,
- $e^{2\phi(x)} - e^{2f(x)} = e^{4\phi(x)}$

Or, To draw the graph of the quadratic function,  $y = x^2 - x$  and state the characteristic of the graph.

8. Answer any three of the following questions:
- $5 \times 3 = 15$

a. Find the value of  $\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3}$ b. Find the maximum and minimum values of  $f(x) = x^3 - 3x^2 - 45x + 13$ 

c. Find the integrals (any two):

(i)  $\int \cos^4 x \, dx$  (ii)  $\int \frac{\tan x}{\ln \cos x} \, dx$

(iii)  $\int \frac{dx}{(x-3)\sqrt{x+1}}$  (iv)  $\int \frac{x \, dx}{(x-1)(x^2+4)}$

d. Find (any Two):

(i)  $\int_0^{\frac{\pi}{2}} \cos^3 x \sqrt{\sin x} \, dx$  (ii)  $\int_0^1 \frac{1+x}{1+x^2} \, dx$

(iii)  $\int_0^4 y\sqrt{4-y} \, dy$  (iv)  $\int_0^{\frac{\pi}{4}} \tan^3 x \sec^2 x \, dx$

1. (b)  $\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{6} & -\frac{5}{6} & \frac{1}{2} \\ -\frac{1}{6} & -\frac{1}{6} & \frac{1}{2} \end{bmatrix}$

2. (a) 1140, 170

3. (a)  $\frac{a^2 - b^2}{2ab}$ ,  $\frac{a^2 + b^2}{a^2 - b^2}$

(c)  $\frac{51}{26}$

5. (a) (3, 2) (b)  $13x - 23 = 0$  (c)  $(\frac{1}{5}, \frac{7}{5})$  (d)  $3x + y = 5$

8. (a)  $\frac{1}{2}$

(b)  $\ln 2 - 1$

(c) (i)  $\frac{1}{4} \left( \frac{3x}{2} + \sin 2x + \frac{1}{8} \sin 4x \right) + c$

(ii)  $-\ln(|\ln \cos x|) + c$

(iii)  $\frac{1}{2} \ln \left[ \frac{\sqrt{(x+1)-2}}{\sqrt{(x+1)+2}} \right] + c$

(iv)  $\frac{1}{5} \ln |x-1| - \frac{1}{10} \ln |x^2+4| + \frac{2}{5} \tan^{-1} \frac{x}{2} + c$

(d) (i)  $\frac{8}{21}$ ; (ii)  $\frac{\pi}{4} + \frac{1}{2} \ln 2$ ; (iii)  $\frac{128}{15}$ ; (iv)  $\frac{1}{4}$